NAG Toolbox for MATLAB

s09ab

1 Purpose

s09ab returns the value of the inverse circular cosine, $\arccos x$, via the function name; the result is in the principal range $(0, \pi)$.

2 Syntax

```
[result, ifail] = s09ab(x)
```

3 Description

s09ab calculates an approximate value for the inverse circular cosine, $\arccos x$. It is based on the Chebyshev expansion

$$\arcsin x = x \times y(t) = x {\sum_{r=0}}' a_r T_r(t)$$

where $\frac{-1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$, and $t = 4x^2 - 1$.

For $x^2 \leq \frac{1}{2}$, $\arccos x = \frac{\pi}{2} - \arcsin x$.

For $-1 \leq x < \frac{-1}{\sqrt{2}}$, $\arccos x = \pi - \arcsin \sqrt{1 - x^2}$.

For $\frac{1}{\sqrt{2}} < x \leq 1$, $\arccos x = \arcsin \sqrt{1 - x^2}$.

For $|x| > 1$, $\arccos x$ is undefined and the function fails.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: $|\mathbf{x}| \leq 1.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

s09ab has been called with $|\mathbf{x}| > 1.0$, for which arccos is undefined. A zero result is returned.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\arccos x \sqrt{1-x^2}} \times \delta \right|.$$

The equality should hold if δ is greater than the ***machine precision*** (δ is due to data errors etc.), but if δ is due simply to round-off in the machine it is possible that rounding etc. in internal calculations may lose one extra figure.

The behaviour of the amplification factor $\frac{x}{\arccos x \sqrt{1-x^2}}$ is shown in the graph below.

In the region of $x = 0$ this factor tends to zero and the accuracy will be limited by the ***machine precision***. For $|x|$ close to one, $1 - |x| \sim \delta$, the above analysis is not applicable owing to the fact that both the argument and the result are bounded $|x| \leq 1$, $0 \leq \arccos x \leq \pi$.

In the region of $x \sim -1$ we have $\epsilon \sim \sqrt{\delta}$, that is the result will have approximately half as many correct significant figures as the argument.

In the region $x \sim +1$, we have that the absolute error in the result, E, is given by $E \sim \sqrt{\delta}$, that is the result will have approximately half as many decimal places correct as there are correct figures in the argument.

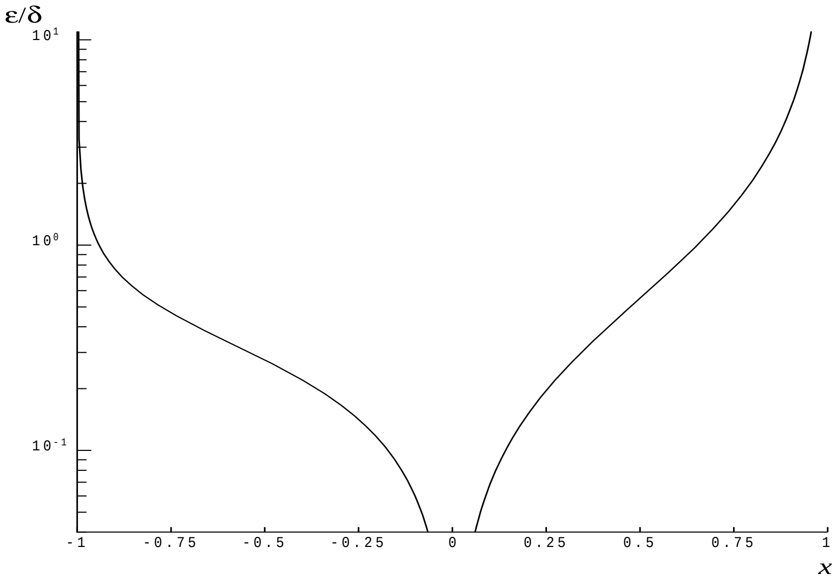

Figure 1

8 Further Comments

None.

9 Example

```
x = -0.5;
[result, ifail] = s09ab(x)
```

```
result =
    2.0944
ifail =
           0
```